FLUID MECHANICS

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Contents

CHAPTER 1
FLUID PROPERTIES

1.1 Fluids

Fluid mechanics is the science that deals with the action of forces on fluids. It can be classified into fluid static and fluid dynamic. Fluid static deals with fluids at rest while fluid dynamic deals with fluids in motion.

The particles of a fluid move easily and accordingly they continuously change their relative position. A fluid can be defined as the substance that continuously deforms (flows) under the action of shear stress no matter how small the magnitude of the shear. The shear force is the force tangent to the surface and this force divided by the area of the surface gives the average shear stress over the area. If the fluid is at rest then the shear stress is zero and Mohr circle for a fluid at rest is a point on the negative side of the normal stress axis. It has a zero radius as shown in figure 1. (Mohr circle is a graphical presentation between the normal stress σ and the shear stress, τ)

On the other hand the solids can resist the shear stress within the elastic limit of the material with no permanent deformation and the deformation is proportional to the applied shear. If shear stress is applied to fluid, it will move. The motion continues as long as the shear is sustained. In contrast to solids, the rate of deformation (deformation per unit time) is proportional to the applied shear (for Newtonian fluids).

Fig. 1 Mohr Circle

1.1.1 Liquids and Gases

Fluids can be classified into liquids and gases. Liquids take the shape of the container. It forms a free surface when the container volume is larger than the liquid volume. A gas completely fills the space it occupies. For liquids, the spacing between molecules is essentially constant while the gas molecules are continuously moving. The spacing of molecules for gases is wider than that for liquids. Solid molecules are arranged in a specific lattice formation and their movements are restricted.

1.1.2 Fluid as a Continuum

Within the present course, the fluid is considered as a continuous substance. Mass, momentum and energy conservations are considered for fluid particles and not for individual molecules. This can be justified by considering the number of molecules in a given volume. The number of air molecules in a cubic millimeter at STP (standard temperature and pressure) is about 10^{17} molecules. Assuming air to be a perfect gas, this number is evaluated as Avogadro's number A multiplied by N number of k molls;

$N = P\,(V)/(\overline{R}T)$ and $A=6.022\ (10\ )^{26}$ molecules / k mol and R is the universal gas constant and equals 8313 J/(k moll ^{o}K), P is the thermodynamic (absolute) pressure in N/m^2 or Pa, V is the volume in m^3 and T is the absolute temperature in ^{o}K.

The above assumption is appropriate as long as the characteristic length is larger than the mean free path of the gas molecules (the average distance a molecule travels between consecutive collisions.). It may be inappropriate for slip flow of rarefied gases.

1.2 Newton Law of Viscosity

Viscosity is defined as a measure of fluid resistance to motion. If a shear stress is applied the fluid will move. The strain rate produced by fluid motion is proportional to the applied shear stress. Consider two layers of fluid separated by distance Δy normal to the relative velocity Δu, produced by a shear stress τ

$\tau \propto (\Delta u / \Delta y)$.

Fig. 2 Variation of shear stress with rate of strain.

The constant of proportionality is known as the coefficient of viscosity μ with dimension $ML^{-1}T^{-1}$or FTL^{-2}. In S I units it can be given by $N.s/m^2$ (Pa.s) or kg/(ms). In the cm-gm-s (cgs) system of units the Poise is usually used. One Poise is 1gm/(cm s) and equals 0.1 Pa.s. The kinematic viscosity ν is defined as μ/ρ with dimension L^2T^{-1} in cgs system of units the Stoke is usually used. One Stoke equals 0.0001 m^2/s.

If the relation between the shear stress and the rate of strain is linear, then the viscosity is constant (μ does not depend on the strain rate) the fluid is said to be Newtonian. Non Newtonian fluids have variable viscosity. Pseudo plastic and dilating fluids are both non- Newtonian fluid. For continuous velocity distribution the rate of strain is given by the derivative (du/dy), see example 1.3 below. Ideal fluids have a negligible viscous effect and can be represented by the strain rate axis, $\mu = 0$. All curves representing fluids must pass by the origin.

The viscosity of liquids decreases as their temperature increases. This can be explained by the decrease of cohesive forces between liquid molecules upon heating. The viscosity of gases increases as the temperature increase. The gas mean free path of molecules increases with heating. The free path is the distance traveled by molecule, between two consecutive collisions.

Fig.3 Variations of viscosity with temperature.

Example 1.1

Find the weight of the shown plate of area 2 m^2 if it falls with uniform speed of 1.25 m/s in oil of kinematic viscosity $\nu = 0.0001$ m^2/s and specific gravity 0.8. The gap is 6 mm. Plate is 1 mm thick. The plate is at equal distances from fixed walls.

Solution

Since the plate moves with constant speed
Then, the weight downward equals the sum of viscous shear forces on both sides and the buoyancy force.

$$\tau = \mu \frac{\Delta u}{\Delta y} = (\nu \rho) \frac{1.25}{0.0025} = 0.08 * 500 = 40 \quad \text{N/m}^2$$

$$Weight = 2 * \tau * A + Volume * \rho g$$

$$= 2(40) * 2 + 0.002 * 9.81 * 800 = 175.7 \quad N$$

Oil bearings are also among other applications on viscosity. It used to reduce friction for shafts rotating at small speed of rotation. The radial clearance between the shaft and sleeve is filled with oil. The following example demonstrates this application.

Example 1.2

A shaft of diameter 10 cm rotates in a fixed sleeve of length 12 cm at a speed of 40 RPM. Sleeve diameter is 102 mm The oil used is SAE 30 W at 20 ^oC with dynamic viscosity µ =4 Pa s. Find the viscous shear stress on the shaft surface, the shear force, the viscous torque and the power consumed in viscous friction.

Solution

Angular speed $\omega = N(2\pi)/60 =$ 4.19 rad/s

Velocity difference $\Delta U = \pi D N /60 = 3.14(0.1)(40) / 60 = 0.21$ m/s

Viscous shear stress $\tau = \mu \; \Delta U/\Delta r = 4(0.21)/0.001 = 840$ N/m^2

Viscous force $F = \tau\,(\pi D L) = 840(3.14)(0.1)(0.12) = 31.7$ N

Viscous Torque $T = F r = 31.7\,(0.05) = 1.58$ N.m

Power consumed in viscous friction = $T\,\omega = 1.58(4.19) = 6.6$ Watt

For the case of thrust bearing, the relative speed varies with radius, and accordingly the shear stress will be function of the radial location. An element of area must be considered, and the torque is calculated by integration over the area covered by the oil. Similarly, for conical bearing, the shear stress is function of radius and integration is required to evaluate the torque and power consumed by viscous friction.

For problems with continuous distribution of velocity with y, the rate of strain equals (du/dy). The following example illustrates this idea for flow between two wide parallel plates.

Example 1.3

The flow of a Newtonian fluid with viscosity 20 Poise between two wide parallel plates separated by a normal distance 2h is given by:

$u = 1.5V[1 - (\frac{y}{h})^2]$ where y is measured from the centerline normal to the walls and V is the average speed . Evaluate the shear stress at 1) the centerline y=0. 2) at the walls y=±h. Use h=0.1 m and V=5 m/s. [1 Poise is gm/(ms)]

Solution

The velocity gradient $\frac{du}{dy} = -3V\frac{y}{h^2}$ it equals zero at the center y=0. It has its maximum magnitude at y =±h. It varies linearly with y

The viscous shear stress $\tau = \mu\, (du/dy)_{y=\pm h}$

$= 20\ (0.1)\ 3(5)/0.1 = 300\ N/m^2$

Similar treatment can be carried out for laminar flow in a circular duct of radius R (Value of Reynolds number ≤ 2300 as will be seen in chapter 5 on flow through pipes) where the velocity profile is given by

$u = 2V[1 - (\frac{r}{R})^2]$, V is the average speed, $V = Q / (\pi R^2)$.

1.3 Pressure

It is defined as the normal compressive force (into the fluid) per unit area. It has units of N/m^2 or Pa. Gage pressure (or pressure) is measured from the atmospheric pressure. It can be positive (for pressure above the atmospheric pressure) or negative for pressures below atmospheric value. Absolute pressure P_{abs} is measured from zero pressure. See figure 3. The absolute pressure reaches zero when an ideal vacuum is achieved, that is when no molecules are left in space. A negative absolute pressure is impossible.

The pressure is a scalar quantity. It does not depend on the direction. Consider the force balance for the shown rectangular prism with dimension Δx, Δy, and unit length normal to the paper.

$\sum F_x = 0$ gives $P_x(\Delta y)(1) - P_s(\Delta s)(1) \sin\theta = 0$

$P_x = P_s$

Similarly, $\sum F_y = 0$ gives

$P_y(\Delta x)(1) - P_s(\Delta s)(1) \cos\theta - \rho g \Delta x \Delta y/2 = 0$

As Δx and Δy both tend to zero and the prism becomes a point $P_y = P_s$. Accordingly at a point in the fluid:

$P_x = P_y = P_s$

1.4 Vapour Pressure

It is the absolute pressure at which liquid continuously evaporates. Vapor pressure is a strong function of temperature. For example the vapour pressure of water at 100 °C is 1 bar, (101300 Pa abs) while it equals 4000 Pa abs at 29 °C.

1.5 Surface Tension σ

It is the force per unit length of the contact line between the liquid and its neighboring surface. According to the theory of molecular attraction, molecules of liquids below the surface act on each other by forces that are equal on all directions. However, molecules near the free surface have greater attraction for each other than they do for molecules below the surface. Molecules at the surface act like a stretched membrane. At the contact line between this surface and adjacent object a net tension is exerted.

With reference to the shown sketch of free body diagrams, the force due to pressure increase is balanced by surface tension force. The pressure increase inside a spherical water droplet in air is given by $2\sigma/r$, inside a soap bubble is $4\sigma/r$ and σ/r in a cylindrical jet.

Example

Evaluate the pressure increase inside a soap bubble of radius 2 mm if the surface tension σ between soap solution and air is 0.075 N/m.

$\Delta p = 4(0.075)/0.002 = 150$ Pa

If the liquid does not wet the surface, the cohesive force between the liquid molecules is more than adhesive force between the liquid and its neighboring surface., the angle of contact in the fluid θ is larger than $\pi/2$ and capillary depression will take place e.g. Mercury with glass surface. On the other hand if the liquid wets the surface, the angle of contact θ is less than $\pi/2$, and capillary rise h will take place e.g. water with glass surface. Equating the vertical component of surface tension force with the weight of fluid column raised by capillary:

$$\sigma (2\pi\, r)\cos\theta = \pi\, r^2 h\gamma$$

$$h = \frac{2\sigma .\cos\theta}{\gamma . r}$$

Where: γ is the specific weight of the liquid in N/m^3. It can be seen that for liquids that do not wet the surface, with $\theta > \pi/2$ and $\cos(\theta)$ is negative, the liquid shows a capillary depression.

<u>1.6 Bulk Modulus of Elasticity K</u>.

If an extra pressure Δp is applied to the fluid, a decrease in volume ΔV takes place. The bulk modulus is defined as the ratio between the increase of pressure and the produced volumetric strain.

$$K = -\frac{\Delta P}{\frac{\Delta V}{V}} = \rho \frac{\partial p}{\partial \rho}$$

The reciprocal of bulk modulus is some time termed the compressibility.

The student can show that for isothermal thermally perfect gas K=P while for isentropic thermally perfect gas it equals γp where γ is the ratio of specific heats at constant pressure and constant volume.

It should be noticed that for incompressible fluid the density is constant and bulk modulus of elasticity tends to infinity since $\Delta V = \Delta \rho = 0$

The speed of sound (small pressure wave) is given by $C = \sqrt{K/\rho}$

$C = \sqrt{(\frac{\partial p}{\partial \rho})_s} = \sqrt{\gamma RT}$ for perfect gas. For air R=287 J/(kg K) and at T= 300 K then C= 347 m/s.

Exercise

1) Choose the most appropriate statement for each of the followings:

a) For a fluid at rest 1) pressure is zero. 2) density is zero. 3) viscosity is constant 4) the shear stress is zero.

b) Newtonian fluids have 1) constant viscosity. 2) variable viscosity. 3)constant density.4) non of these.

c) Ideal fluid has 1) negligible viscous effect. 2) constant density.3) constant viscosity. 4) Large viscosity.

e) The pressure increase due to surface tension σ inside a spherical water droplet in air is 1)σ/r. 2) 2 σ/r. 3) 3σ/r 4) 4 σ/r.

f) For incompressible fluid the bulk modulus of Elasticity K tends to 1) zero. 2) P. 3) γP. 4) infinity.

2) A block weighting 500 N and having an area of 0.2 m^2 slides down an inclined plane with a constant velocity 2 m/s. An oil gap between the block and the plane is 0.03 mm thick, the inclination of the plane is 30^o to the horizontal. Find the viscosity of the lubricating film.

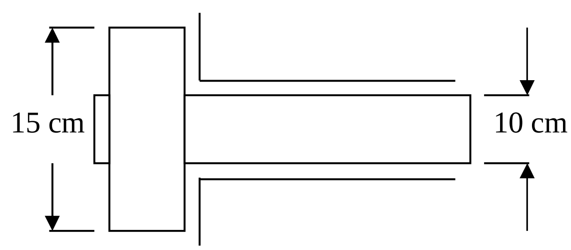
3) A shaft of 15 cm in diameter rotates at 1800 rpm inside a bearing of 15.05 cm in diameter and 30 cm long. The uniform space between them was filled with oil of viscosity μ = 0.018 kg/(m.s). What is the power required to overcome viscous resistance in the bearing?

4) A piston of 6.9 cm diameter rotates concentrically inside a cylinder 7 cm diameter. Both the piston and cylinder are 8 cm long. Find the tangential velocity of the piston if the space between the cylinder and the piston was filled with oil of viscosity 2.35 Poise and the torque of 1.37 Nm is applied t overcome the viscous resistance.

5) A piston 7.5 cm in diameter and 10 cm long moves vertically in an oil dash-pot, the uniform clearance between piston and dash-pot wall being 0.12 cm. The piston falls under its own weight with uniform velocity through 2.5 cm in 50 sec. with an extra weight of 136 gm on the top of the piston it falls through 2.5 cm with uniform speed in 43 sec. Find the value of coefficient of viscosity μ of the oil in poise.

6) A water bug is suspended on the free surface of a pond by surface tension. The bug has six legs, and each leg is in contact with water over a length of 1 cm. What is the maximum mass in gm for the bug to avoid sinking? $\sigma = 0.073$ N/m.

7) The trust of a shaft is taken by collar bearing provided with a film of oil of uniform thickness between the surface of the collar and the surface of the bearing. Internal and external diameters of the collar are 10 cm and 15 cm respectively. If the thickness of oil film separating the two surfaces is 0.25 mm and the oil viscosity is 0.91 N.s/m^2, find the power lost to overcome viscous resistance when the shaft rotates at 300 rpm.



8) Evaluate the capillary rise of water (σ =0.074 N/m) between two vertical glass plates spaced 0.5 mm apart.